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APPLICATION OF ION THRUST MOTORS IN ATTITUDE AND POSITION CONTROL OF SATELLITES*

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ABSTRACT

The first contemplated use of ion propulsion is in the attitude control and station keeping of a synchronous satellite. This paper reviews the major forces (e.g., solar-lunar attraction, earth's triaxiality, solar pressure, etc.) which tend either to perturb the satellite orbit or to modify its orientation.

The mission constraints which affect the engine system design (such as the impulse requirements associated with vernier orbit corrections, the magnitude of disturbance torques, and the required thrusting directions) are presented. An evaluation is made of the trade-offs among such critical mission and system parameters as attitude accuracy, average power utilization, energy storage requirements, duty cycle, thrust level, thrust intervals, and satellite mass and moments of inertia. From these trade-offs the capabilities and requirements of both the attitude control and station-keeping systems can be determined for application over a wide range of satellite weights.

In addition, an optimization and preliminary design of an ion engine attitude control and station-keeping system are given. Such system parameters as power level, thrust level, specific impulse, and weight, are specified.

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1.0. INTRODUCTION

As a result of the rapid progress made in the field of ion propulsion during the past few years, it is now possible to consider ion propulsion systems for near future missions. Research and development programs on ion thrust devices have met with considerable success and several types have already passed the laboratory demonstration stage. In the next few years a series of space tests will be carried out to demonstrate the feasibility of these devices as space propulsion systems. Upon successful conclusion of these tests, complete ion propulsion systems can be made available for space applications.

The first contemplated use of ion thrust devices is in the control of synchronous earth satellites. At the present time an ion propulsion attitude control and station-keeping system is under development for NASA at the Hughes Research Laboratories. (1) This system, which is designed to demonstrate the satellite control capabilities of electric propulsion, can, for example, hold the orientation of a 550-lb synchronous satellite to $\pm 0.5^{\circ}$ in three axes and maintain its position above a point on the surface of the earth to $\pm 0.1^{\circ}$ in both longitude and latitude over a period of three years. Mission-ready systems will be available after completion of orbital flight tests now planned for 1965. (2)

The design and operating mode of such an attitude control and station-keeping system depend primarily on the requirements of the mission, the configuration of the satellite, and the characteristics of the ion thrust device. In order to optimize this system, therefore, it is necessary to know in detail the magnitude and nature of the major forces which tend either to perturb the satellite orbit or to modify its orientation. The total impulse, thrust directions, and possible thrust modes can thus be derived. In addition, consideration must be given to such critical thrust device parameters as ionizer warm-up energy and power efficiency. With these data, the trade-offs between propulsion system and mission parameters (e.g., average power utilization, energy storage requirement, duty cycle, thrust level, thrust interval, attitude and station-keeping accuracies, and satellite mass and moments of inertia) can be evaluated. The control system design can then be optimized for each application and its operating characteristics can be predicted.

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This paper has three purposes: (1) to review the major forces which tend either to perturb the satellite or modify its orientation; (2) to provide the trade-offs from which the capabilities and requirements of both the attitude control and station-keeping systems can easily be determined for application for a wide range of satellites; and (3) to present a preliminary design of an ion engine attitude control and station-keeping system.

2.0. MISSION ANALYSIS

2.1. Attitude Control

The attitude control system must hold the orientation of a satellite about three axes to a specified angular accuracy for a period of years. It must be designed to operate effectively in the presence of disturbance torques which arise from such natural effects as solar radiation, gravity, magnetic fields, and micrometeorite impact and such internal effects as thrust misalignment, moving parts, and gas leakage. For the sake of reliability the system design must be simple, and yet it must be versatile enough to counteract unbalanced torques ranging from some maximum value to zero. At the synchronous orbit altitude, the major external disturbance torque acting on a satellite is unbalanced solar radiation pressure. Since these torques are generally a function of vehicle configuration, they are rather difficult to predict accurately. However, by using a rather simple model, along with the equations developed in Reference (3), the solar pressure torques T can be approximated by the following equation:

$$T = VA\ell(1 + r) \tag{1}$$

where

$V \equiv$ solar pressure at normal incidence (no reflection)

$A \equiv$ effective area normal to sun

$\ell \equiv$ effective moment arm from center of solar pressure to vehicle center of mass

$r \equiv$ average reflectivity of surfaces exposed to sun.

(A better model is given in Reference (3); however, evaluation of solar torques is still difficult, since the effective centers of pressure of each surface of the vehicle must be estimated.) For example, a satellite in the 500 lb class could probably be constructed so that the unbalanced torque would be on the order of 100 dyn-cm.

The largest internal torques due to the control system itself will be caused by the thrust misalignment of the station-keeping engines and by the uncertainty in the position of the center of gravity. In order to keep these torques to less than 10% of those from the unbalanced solar pressure, the station-keeping thrust vector, in certain cases, must be aligned to an accuracy greater than 0.02° and the center of gravity of the vehicle determined to within 0.01 in. For a combined attitude control and station-keeping system, therefore, the alignment of the thrust vector through the precise center of gravity of the satellite must be given very serious consideration.

2.2. Station Keeping

Two sources of perturbing forces dictate the control system requirements for maintaining a satellite stationary in a 24-hour orbit: the gravitational attraction of the sun and moon, and the triaxiality of the earth. The magnitude and nature of these perturbations will determine the design and mode of operation of the ion propulsion station-keeping system.

2.2.1. Sun-Moon Effect

Of all the celestial bodies, only the sun and moon produce significant perturbing effects on an earth satellite. The force of attraction from these two bodies can be divided into three components: two, the radial and tangential, lie in the equatorial plane, and the third lies normal to this plane. The radial and tangential components produce cyclic oscillations in satellite radius and longitude. It can be shown that these oscillations have a small maximum amplitude throughout each satellite orbital period and that at the end of that time both the change and rate of change of satellite radius and longitude will have assumed their initial zero values (e.g., see Fig. 1). Hence, no corrective thrust is required to counteract the radial or tangential force components.

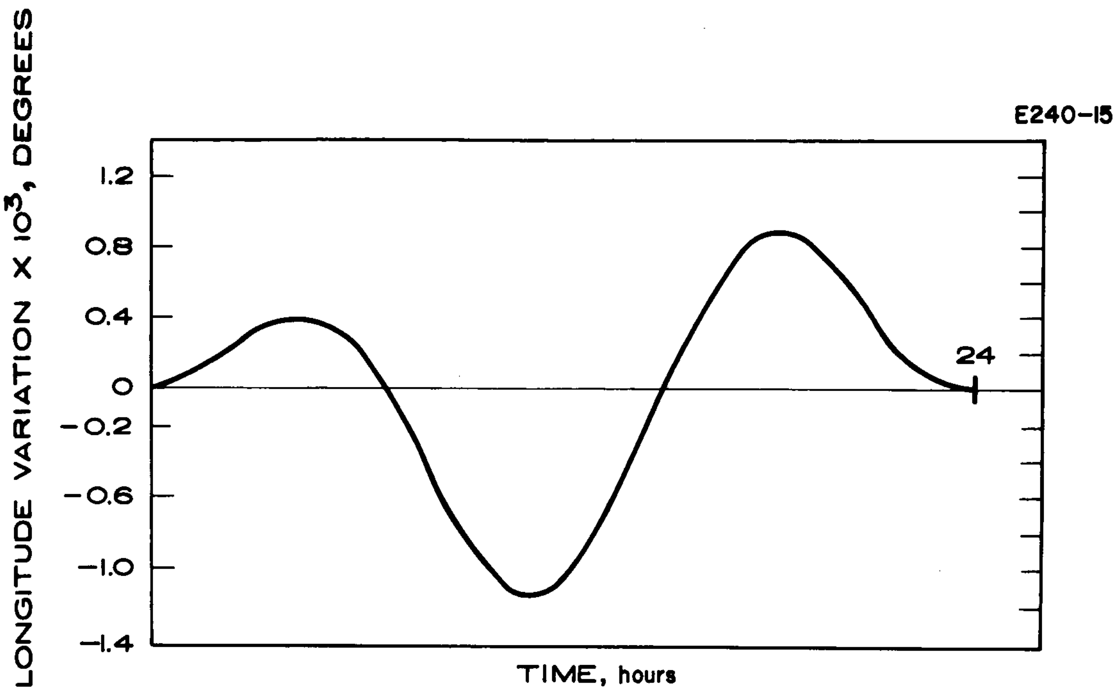

Fig. 1. Typical daily longitude variation of 24-hour satellite due to solar and lunar perturbations.

The component of force normal to the equatorial plane (referred to as the north-south or "Z" component) will, on the other hand, cause an oscillation which results in an increase of satellite orbit inclination at the initial rate of almost 1° per year. The amplitude of this oscillation will grow to a maximum of about 20° in about 40 years. (4) If the satellite is to remain stationary in the equatorial plane, it is necessary to correct this perturbation. The optimum mode of correction and the magnitude of thrust required can be determined by solving the appropriate differential equation, taking corrective thrust into consideration. (5)

The equation of motion of a satellite relative to the earth can be found by considering the attractive force equations. Referring to Fig. 2, the acceleration of the earth with respect to the sun and moon is

$$\vec{a}_E = - K_s^2 \frac{\vec{R}_s}{R_s^3} - K_m^2 \frac{\vec{R}}{R^3} \tag{2}$$

where

$K_s^2 \equiv$ sun's gravitational constant

$K_m^2 \equiv$ moon's gravitational constant

$\vec{R}_s \equiv$ vector of magnitude R_s drawn from the center of the sun to the center of the earth

$\vec{R} \equiv$ vector of magnitude R drawn from the center of the earth.

Similarly, the acceleration of the satellite with respect to the sun and the moon is

$$\vec{a}_{sat} = - K_s^2 \frac{(\vec{R}_s + \vec{r})}{|\vec{R}_s + \vec{r}|^3} - K_m^2 \frac{(\vec{R} + \vec{r})}{|\vec{R} + \vec{r}|^3} \tag{3}$$

where $\vec{r}$ is a vector of magnitude r drawn from the center of the earth to the center of the satellite.

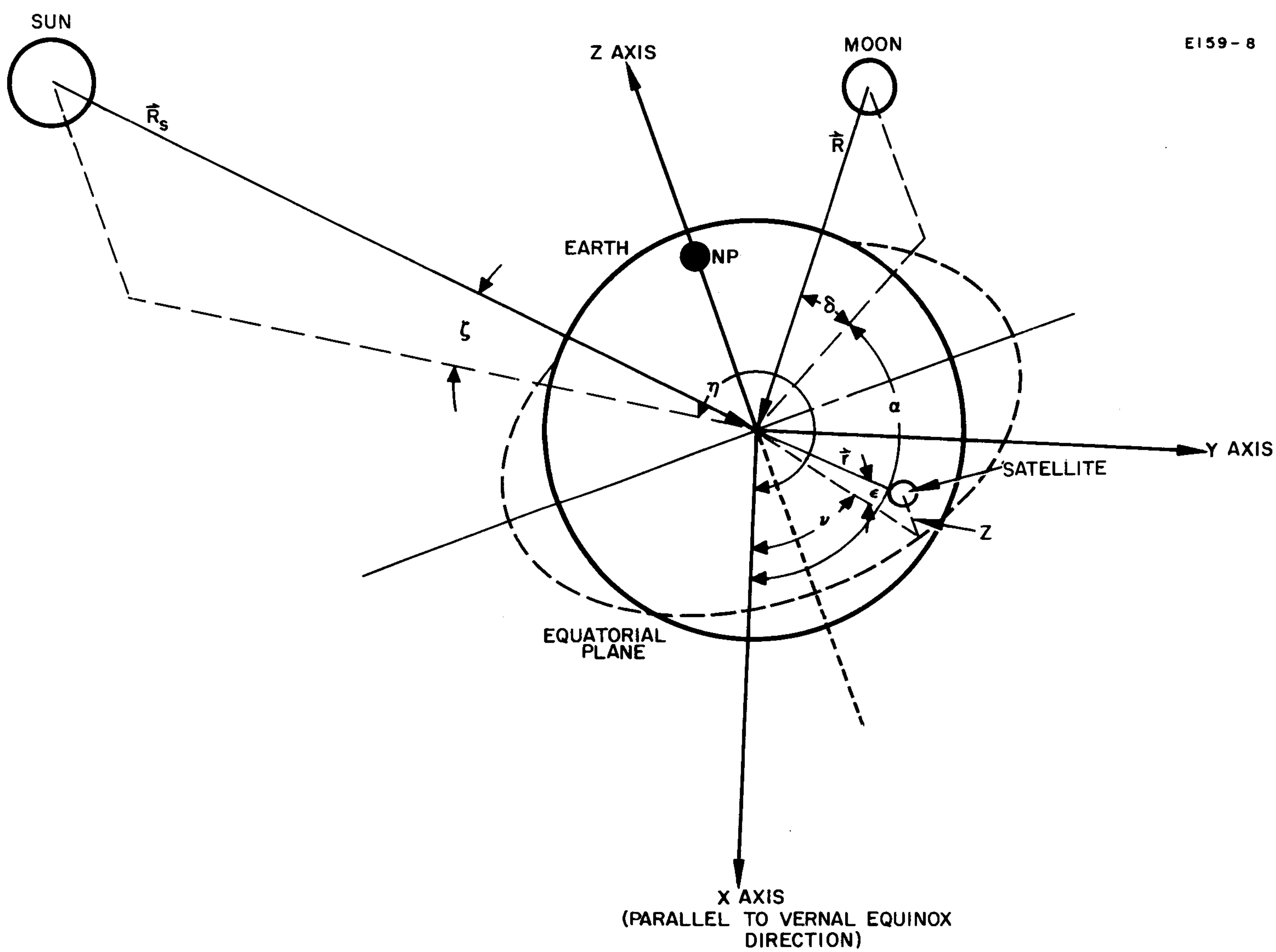

Fig. 2. Geocentric coordinate system.

Therefore, the total acceleration of the satellite $\ddot{\vec{r}}$ with respect to the earth due to the sun, moon, and earth can be written:

$$\ddot{\vec{r}} = \vec{a}_{sat} - \vec{a}_E - \frac{K^2}{r^3}\vec{r} \quad . \tag{4}$$

Finally, by combining Eqs. (2), (3), and (4), it can be seen that

$$\ddot{\vec{r}} + \frac{K^2}{r^3}\vec{r} = -K_m^2\left[\frac{\vec{R}+\vec{r}}{|\vec{R}+\vec{r}|^3} - \frac{\vec{R}}{R^3}\right] - K_s^2\left[\frac{\vec{R}_s+\vec{r}}{|\vec{R}_s+\vec{r}|^3} - \frac{\vec{R}_s}{R_s^3}\right] . \tag{5}$$

Equation (5) is the general differential equation, in vector form, which describes the satellite's orbital motion in the presence of a spherical earth, the sun, and the moon. By dividing the vector equation into its components, the equation which defines the latitudinal motion of the satellite is found to be

$$\ddot{Z} + \frac{K^2}{r^3} Z = \frac{K_m^2}{R^3}\left[3 \sin\delta \cos\delta \cos(\nu - \alpha)\right]r$$
$$+ \frac{K_s^2}{R_s^3}\left[3 \sin\zeta \cos\zeta \cos(\nu - \eta)\right]r \tag{6}$$

where

$Z \equiv$ perpendicular distance from the equatorial plane

$\delta \equiv$ declination of the moon

$\nu \equiv$ right ascension of the satellite

$\alpha \equiv$ right ascension of the moon

$\zeta \equiv$ declination of the sun

$\eta \equiv$ right ascension of the sun.

The solution to Eq. (6) is shown in graphical form in Fig. 3, where the uncorrected Z-oscillation is given for a 48-day interval during which the sun passed through a point of maximum perturbation. The maximum moon effect occurs every 13.6 days. Fig. 3(c) illustrates the total solar-lunar effect on the orbital inclination of a satellite for the 48 day period, starting November 26, 1965. The inclination at the end of this time is 0.16°. The effects of the solar and lunar perturbations are shown seperately for the same period in Figs. 3(a) and 3(b), respectively. From these curves the total change per year in orbital inclination can be determined. The sun's contribution is found to be 0.30°, whereas that of the moon is 0.64°, for a total of 0.94° per year. From this result the minimum amount of north-south station-keeping propulsion required daily can be calculated.

In order to counteract the change in inclination and maintain the satellite in the equatorial plane, velocity must be added to the satellite in a direction normal to the orbit plane. Because of the gyroscopic nature of the satellite in its orbit, the orbit will precess about an axis directed through the point of applied torque and normal to the thrust. If the satellite is to be precessed into the equatorial plane, corrective thrust can be applied most efficiently at the nodal points, i.e., satellite crossings of the equatorial plane. (It will be shown later that this is not necessarily the optimum thrust mode for an ion propulsion system.) For a 24-hour satellite these crossings occur twice daily. As an example, assuming a thrust mode of two nodal firings per day, alternating in a north-south direction depending on whether the satellite is crossing an ascending or descending node, the total impulse required each day to correct the total solar-lunar perturbations (0.94° per year) on a 550 lb satellite is 7.8 lb-sec. If 1.5 mlb station-keeping engines are employed, thrust must be applied for 43.5 min per firing. (Because the required thrust time when correcting twice daily at the nodes can be determined analytically, a reference correcting mode can be established to which the effectiveness of other thrusting sequences may be compared.)

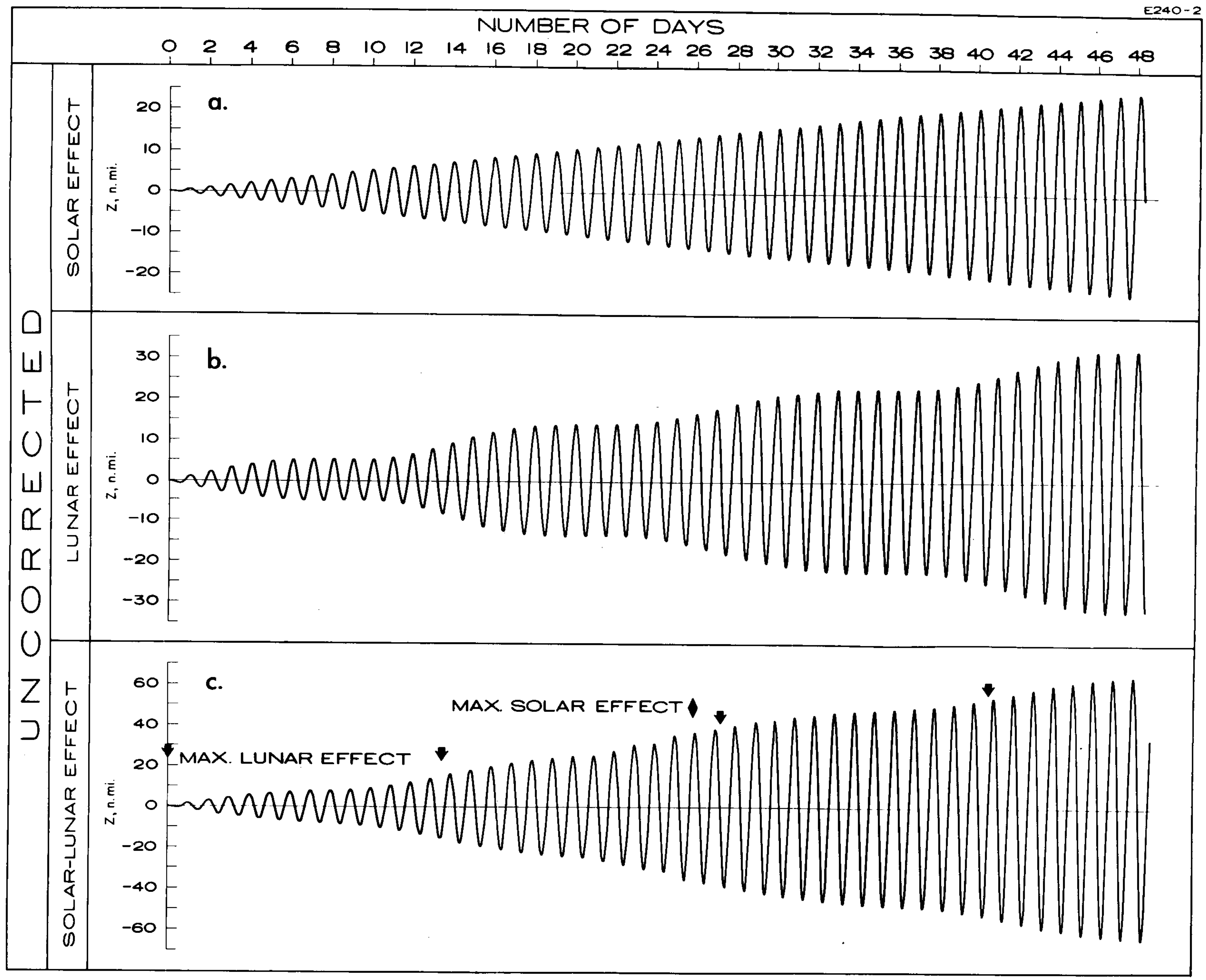

Fig. 3. Uncorrected latitudinal satellite motion.

The variation in latitude, as defined by the parameter Z of a 550 lb synchronous satellite under the influence of the solar-lunar perturbation and several different thrust sequences is shown in Fig. 4. The data presented are for a 550-lb satellite using 1.5 mlb station-keeping engines (except in the cases where continuous thrust is employed); however, the results may be extrapolated to other size vehicles. The corrective thrust is applied in a direction to decrease the satellite's velocity component normal to the equatorial plane. Figure 4(a) shows the effect of thrusting 43.5 min twice daily at the equatorial crossing (nodes). Since this thrust sequence has been shown to be effective, it will be used as the reference. In this case the satellite is held to an inclination of 0.027°.

As will be shown later, the weight of the energy storage system required to operate an ion propulsion control system in a pulsed mode is determined primarily by the length of thrust intervals per orbit correction. If it were possible, therefore, to fire more often each day, resulting in a shorter thrust period, a considerable reduction in system weight could be realized. Furthermore, if it were possible to correct continuously at sufficiently low thrust levels to run directly off the primary power source, the energy storage system could be eliminated. For this reason, other thrusting modes must be considered if the ion propulsion station-keeping system is to be optimized. The length of the thrust interval will vary according to the scheme being considered. Figure 4(b) shows the result of thrusting each time the satellite crosses a node and exactly 3 hours afterward. Each firing interval for this case must be 25 min long to accomplish the same result as firing twice daily at the nodal points. This represents a total daily thrust time of 104 min, corresponding to an impulse of 9.0 lb-sec.

Figure 4(c) illustrates the effectiveness of applying thrust 3 hours before and 3 hours after a node crossing as well as at the node, a total of six firings per day. The thrust time per firing required to control the vehicle was found to be 18 min for a total thrust time of 108 min per day and an impulse of 9.7 lb-sec.

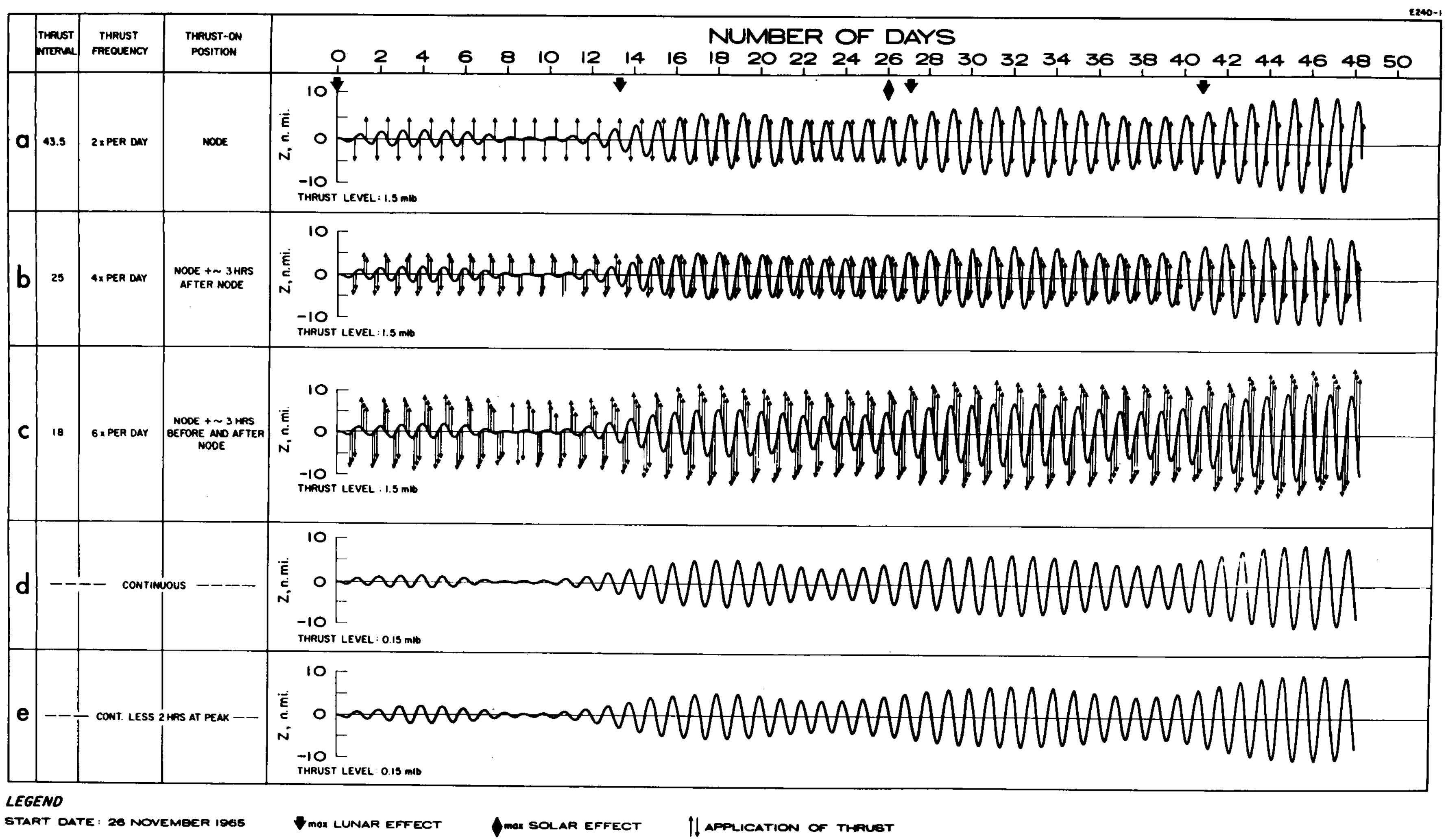

Fig. 4. Latitudinal control using various thrust sequences.

It is apparent, therefore, that the number of firings executed each day can be increased and that the individual thrust periods will be shortened as a result. Another possible correction mode to be considered is continuous thrusting at lower thrust levels. Figure 4(d) shows the results of a continuous 0.15 mlb thrust. This level is seen to be sufficient to control the satellite. However, thrust applied near and at the peaks of the oscillations is essentially useless due to the gyroscopic effect. Therefore, a correction curve again using 0.15 mlb continuous thrust, except for a 2-hour period when passing each peak, is shown. Figure 4(e) illustrates the successful controlling of the satellite by this technique. The total impulse required, however, has increased to 10.8 lb-sec.

2.2.2. Triaxiality Effect

A second major perturbing force on the orbit of a synchronous satellite results from the nonspherical nature of the earth. The three major deviations in the earth's sphericity are the oblateness or the bulging out at the equator, the pyriformity or pear shapedness due to the difference in the compressed heights of the North and South Poles, and the triaxiality or ellipticity of the earth's cross section. The first two of these deviations are a function of latitude only, while the third is dependent on longitude as well.

The only significant nonspherical earth effect on a synchronous satellite is that resulting from the triaxiality. The triaxiality effect causes a satellite to oscillate in an east-west direction with an amplitude of as much as 90° and with a period of up to 3 years. (4) The initial direction and magnitude of these oscillations depend only on the longitude of the desired satellite station. These perturbations on a satellite orbit can be quantitatively described by considering the exact form of the earth's gravity potential which includes terms representing the various deviations from a sphere. (6) The magnitudes of these deviations have been obtained through the use of artificial earth satellites.

The gravitational field of the earth can be expressed by the potential (referring to Fig. 5),

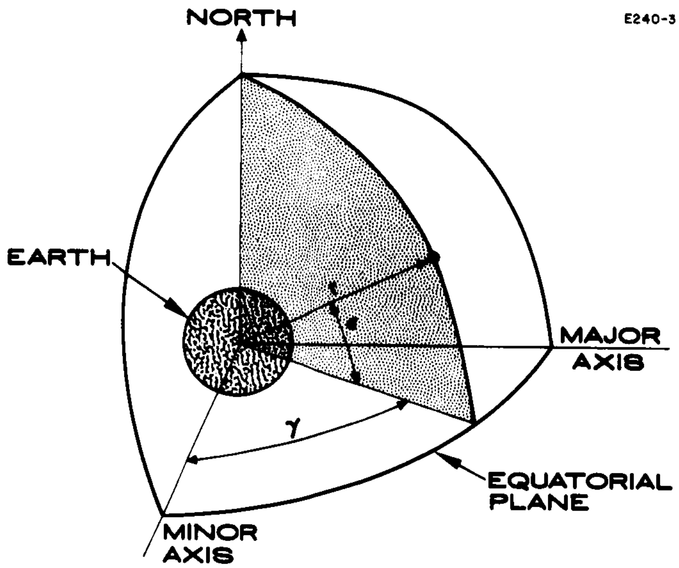

Fig. 5. Geocentric coordinate system.

$$U = \frac{K^2}{r}\left[1 + \frac{A}{r^2}(1 - 3\sin^2\epsilon) + \frac{B}{r^3}(3\sin\epsilon - 5\sin^3\epsilon)\right.$$

$$\left. + \frac{C}{r^4}(3 - 30\sin^2\epsilon + 35\sin^4\epsilon) - \frac{T}{r^2}(\cos^2\epsilon\cos 2\gamma)\right] \quad (7)$$

where

$K^2 \equiv 8.11 \times 10^{11}$ n. mi^3/hour^2

$r \equiv$ radius of satellite orbit in nautical miles

$A \equiv 64.18 \times 10^2$ n. mi^2

$B \equiv 49.35 \times 10^3$ n. mi^3

$C \equiv 29.87 \times 10^6$ n. mi^4

$T \equiv 78.28$ n. mi^2

$\epsilon \equiv$ declination of the satellite

$\gamma \equiv$ longitude of the satellite as measured from the equator's minor axis.

This equation assumes that the earth's surface can be described by a series of spherical harmonic terms, each term representing one surface harmonic of the earth's shape. The first term in Eq. (7) is derived from the spherical part of the earth's mass distribution, the second term from the oblateness, the third term from the pyriformity, the fourth term from the minor deviation in the hemispherical mass distribution, and the fifth term from the triaxiality.

The radial, normal, and tangential accelerations on a satellite in an equatorial orbit due to the nonspherical potential are given by

$$a_r = \left.\frac{\partial U}{\partial r}\right|_{\epsilon = o} = -\frac{K^2}{r^2}\left[1 + \frac{3A}{r^2} + \frac{15C}{r^4} - \frac{3T}{r^2}\cos 2\gamma\right] \quad (8)$$

$$a_\epsilon = \frac{1}{r} \left.\frac{\partial U}{\partial \epsilon}\right|_{\epsilon = o} = \frac{3K^2 B}{r^5} \tag{9}$$

$$a_\gamma = \frac{1}{r \cos \epsilon} \left.\frac{\partial U}{\partial \gamma}\right|_{\epsilon = o} = \frac{2K^2 T}{r^4} \sin 2\gamma \quad . \tag{10}$$

If a_γ is positive toward the east, γ is related to longitude λ by

$$\gamma = \lambda_o - \lambda \tag{11}$$

where $\lambda_o = 56^\circ\ 51'$ E and λ is the longitude measured eastwardly. Examination of Eqs. (8) and (9) shows that the perturbing accelerations in the radial and normal direction are negligible and can be made zero by proper initial injection. Equation (10) shows that the tangential acceleration equals zero when γ equals 0°, 90°, 180°, 270°; from Eq. (11), at longitudes of 56° 51' E, 123° 9' W, 146° 51' E, and 33° 9' W, the first values of γ represent positions of stable equilibrium (the minor axis) and the latter two positions of unstable equilibrium (the major axis). If a satellite is to remain at a longitude other than 56° 51' E or 123° 9' W, some form of station-keeping propulsion is required. The maximum total impulse per unit time required to hold the east-west position of a synchronous satellite occurs at longitudes 11° 51' E, 101° 51' E, 78° 9' W, and 168° 9' W. The tangential acceleration a_γ in this case is 2.22×10^{-7} ft/sec^2, corresponding to a velocity increment ΔV of 1.92×10^{-2} ft/sec/day. A 1.5-mlb station-keeping engine, for example, would have to thrust 3.64 min/day in the direction opposite to the perturbing acceleration in order to maintain a synchronous satellite at a longitude of maximum triaxiality effect. Figure 6 shows the variation of daily velocity requirement superimposed on a projection of the earth.

The optimum mode in which the correcting thrust takes place depends primarily on the nature of the effect of the perturbation on the satellite motion. In the case of triaxiality, the perturbing force tends to cause the satellite

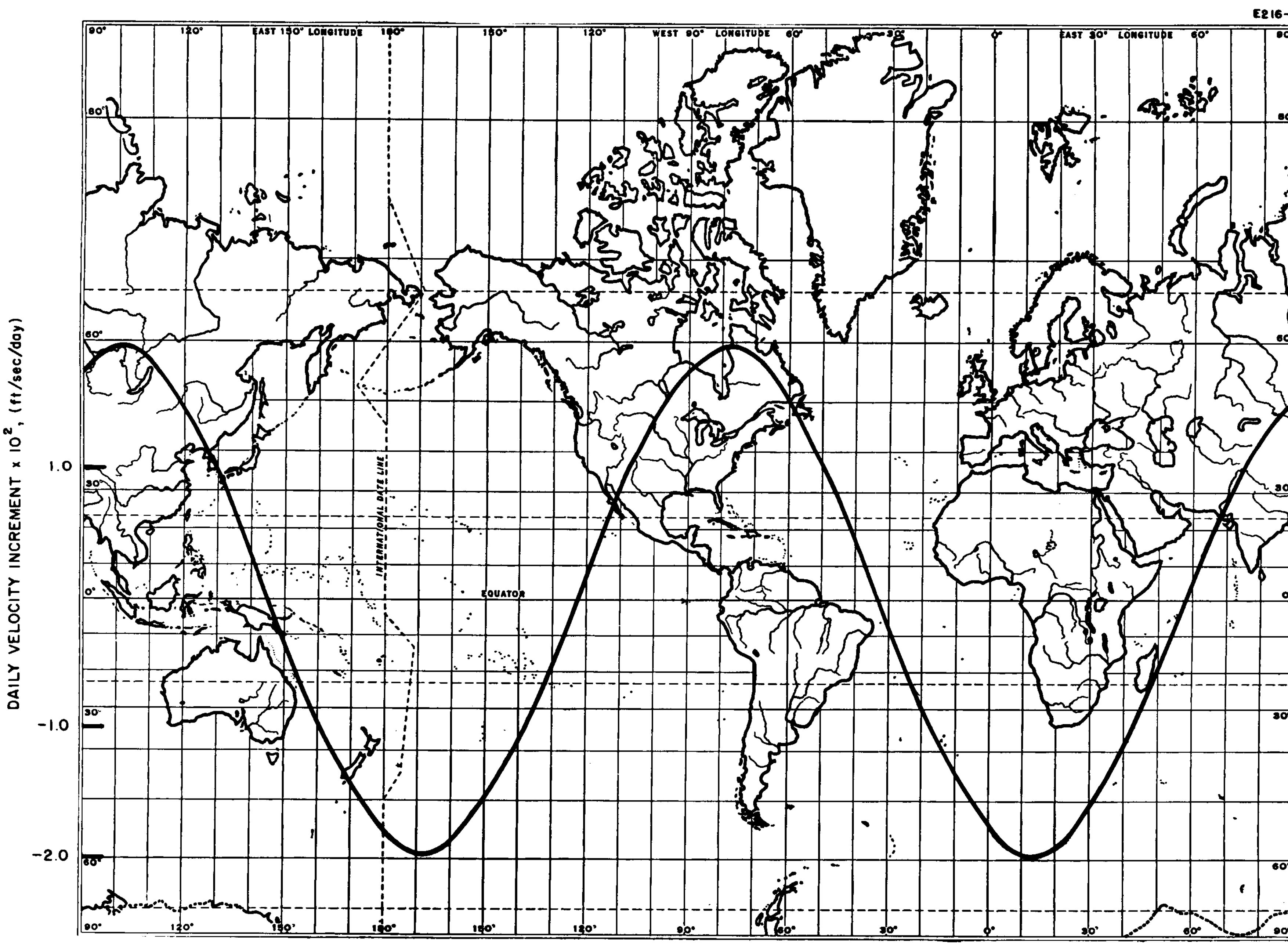

Fig. 6. Daily velocity increment versus station longitude.

to spiral outward away from or inward toward the earth depending on the station longitude. Figure 7 illustrates the actual corrected motion in the equatorial plane of a satellite, assuming the perturbing force is acting in the direction of the satellite orbital velocity. The correction sequence in this case has a period of 24 hours, beginning at a point A which lies on a circular orbit of radius r_1. The radius r_1 is related to the radius r of the synchronous orbit by

$$r_1 = r - \Delta r \tag{12}$$

where $2\Delta r$ is a change in satellite altitude after 12 hours of perturbation. After 12 hours of continuous perturbing force the satellite will have spiraled out to an orbit of radius $r + \Delta r$ (point B) it will have gained an energy corresponding to 9.6×10^{-3} ft/sec (i.e., one half of the total ΔV per day). If this energy is now removed by an impulse from the station-keeping engines, the satellite will be placed on a transfer ellipse whose total energy is equal to that of the original circular orbit and whose perigee would be $r_1 - 2\Delta r$ (point C). Since, however, the perturbing force increases the satellite orbit $2\Delta r$ during the transfer, the actual perigee will be equal to r_1 and the energy gained equivalent to 9.6×10^{-3} ft/sec. If this velocity increment is once again removed by the station-keeping engine, the satellite will assume its initial position and velocity values at point A. Therefore, the longitudinal position of the satellite will oscillate about the synchronous point with a period of one day and with a net change of zero.

In the example above, the total thrust time per day required to counteract a velocity increment of 1.92×10^{-2} ft/sec is 3.64 min. Each thrust interval must be 1.82 min. In the case where the perturbing force is in the direction of a satellite motion, the corrected thrust is applied each time so that the orbital velocity is decreased. This correction mode requires only one station-keeping engine to counteract the triaxial or east-west perturbation.

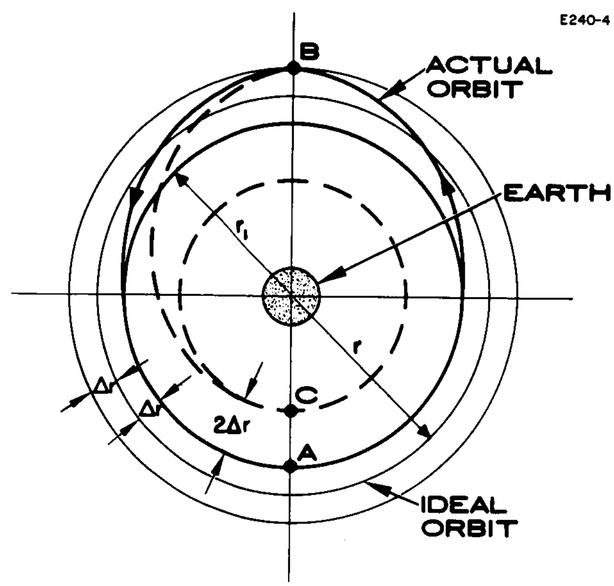

Fig. 7. Corrected longitudinal satellite motion.

2.3. System Constraints

In order to provide three-axis attitude control and station-keeping for a 24-hour satellite, it is necessary that thrust be directed in at least nine directions (see Fig. 8). Attitude control of each axis is accomplished by thrusting in two opposite directions at the end of a moment arm, providing control over both positive and negative angular displacements. Both north-south and east-west station keeping require a thrust vector through the center of gravity of the satellite. The latitudinal station keeping is provided by thrusting alternately in the north and south direction perpendicular to the satellite orbit. The longitudinal correction is accomplished by thrusting in one direction (either east or west depending on the station longitude) tangent to the orbit. For example, at $11^{\circ}\ 51'$ W longitude the perturbing acceleration is eastward, causing the satellite to gain altitude and develop an apparent drift westward. The corrected thrust in this case is applied in a westward direction against the orbital motion of the satellite.

It is possible to meet the nine thrust direction requirement using either nine ion engines (one for each direction) or a smaller number by employing either a gimbaling system for thrust vector position or electrostatic deflection of an ion beam. The choice must be made on the basis of the power and weight constraints necessarily imposed on an ion propulsion attitude control system.

3.0. SYSTEM PERFORMANCE

The design of an ion propulsion system for primary vehicle attitude control and station keeping will be governed by the trade-offs between the mission requirements, the vehicle configuration, and the engine system characteristics. These trade-offs are somewhat unique for electric propulsion. For a mass expulsion system, thrust duty cycle (or propellant requirement) is generally the critical performance parameter. However, in the case of the high specific impulse electric propulsion system the most critical performance parameter is the average power consumption. For this reason, all the performance criteria discussed are related to the average power consumption.

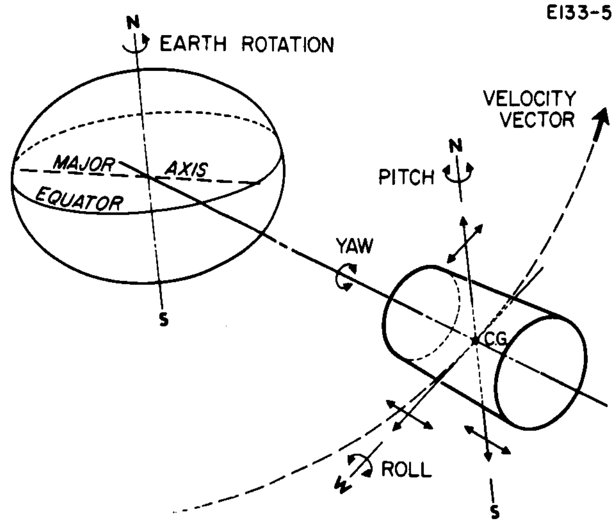

Fig. 8. Required thrusting directions for three-axis attitude control and station keeping of a 24-hour satellite.

3.1. Attitude Control

One of the major advantages of an ion propulsion attitude control system is the precise control of the minimum impulse bit afforded by the ion thrust device (e.g., thrust is terminated instantaneously by voltage shutdown and valve closure). Because of this capability the angular deviation and the angular rate of a satellite as well as the propellant utilization can be held to a minimum.

The principal trade-offs involved in the attitude control performance are those between attitude accuracy desired, average power available, and fixed vehicle parameters such as moment of inertia and maximum expected disturbance torque. It will be shown that these trade-offs are critical in determining the attitude control performance achievable and that, in some cases, certain performance requirements must be relaxed in order to meet other more critical requirements. For the purpose of the performance analysis, the following assumptions are made:

1. A single axis analysis is applicable since cross coupling torques are small. The results of the single axis analysis can then be applied to the three axis case by assuming three similar single axis systems.
2. The thrust is essentially constant during the thrusting periods.
3. The attitude references are initially acquired by an auxiliary method; hence the ion propulsion attitude control system will operate in a limit cycle or near limit cycle mode during the vehicle lifetime.

The limit cycle operation in the phase plane is shown in Fig. 9 for two disturbance torques T_d: (a) $T_d = 0$, hard limit cycle, and (b) $T_d \neq 0$, soft limit cycle. In a hard limit cycle operation (a), thrust is applied at both sides of the prescribed angular limit, whereas the soft limit cycle (b) requires thrust only at one extremity. The latter situation occurs when a disturbance

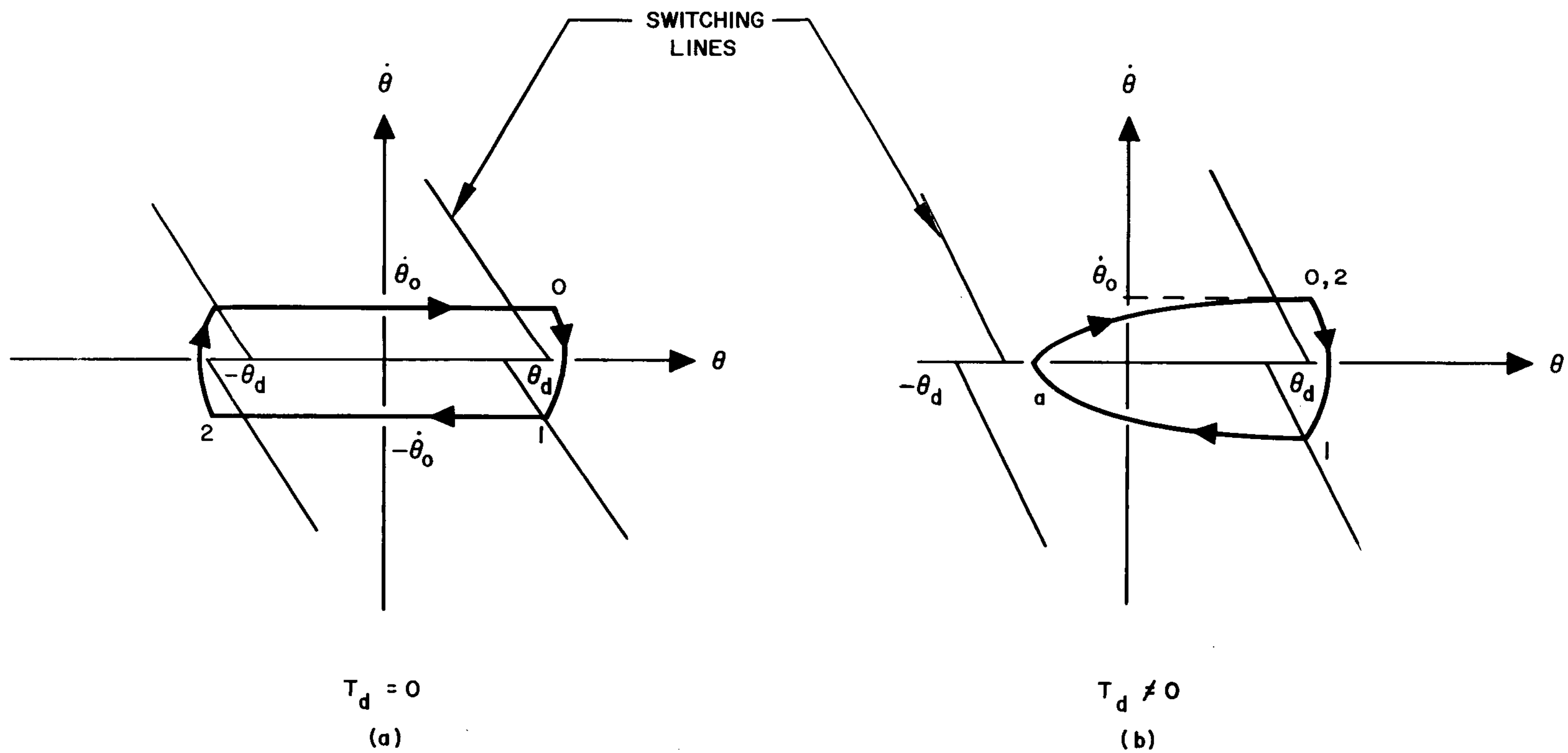

Fig. 9. Phase plane diagram of system in limit cycle operation.

torque is large enough to decelerate the vehicle and reverse its direction before the opposite switching line (proper error signal) is reached. It can be shown that the system will go into a hard limit cycle when

$$0 \le T_d \le T_d\,(\mathrm{max})/4$$

and a soft limit cycle when

$$T_d(\mathrm{max})/4 < T_d \le T_d(\mathrm{max})$$

where $T_d(\mathrm{max})$ is the design maximum for the system. If the minimum impulse bit of a system can be controlled so that a soft limit cycle is the normal mode of operation, it provides position and rate control at the minimum expenditure of corrective impulse. This fact, along with the high specific impulse (propellant utilization efficiency) of an electric thrust device, makes the ion propulsion system an attractive method for controlling the orientation of a space vehicle.

The most stringent constraint governing the design of an attitude control loop is the average power utilization requirement during the nonthrusting periods. Due to the unique nature of the cesium surface contact ion engine (the only type considered here) power must be supplied to heat the ionizer for some period before each firing. (In general, since the duty cycle associated with an attitude control system is low, warming the ionizer for each firing is more economical power-wise than continuous heating at the operating temperature.)

Since the average power available on a vehicle may be less than that required to heat the ionizer, a sufficient time must be allowed on the average between successive engine firings in order that this energy may be stored, for instance, in batteries. Hence, the coast time from point 1 to point 2 (or point 1 to point a to point 2) for $T_d = 0$ must be

$$t_{12} \geq \frac{P_w t_w}{P_a - P_c} \tag{13}$$

where

$P_w \equiv$ warmup power

$t_w \equiv$ warmup time

$P_a \equiv$ average power available

$P_c \equiv$ continuous power.

This constraint on a coast time will then determine the minimum thrusting time t_{01} which will allow a coast of at least

$$\frac{P_w t_w}{P_a - P_c} \quad .$$

For the case of $T_d \neq 0$, the relationship between the coast time and the thrust time in the limit cycle is

$$t_{01}/t_{12} = T_d/T_c \tag{14}$$

provided that the limit cycle is "one-sided," that is, the thrust is always applied in one direction as shown in Fig. 7. Then, from Eqs. (13) and (14) the minimum thrust time must be

$$t_{01} = \frac{T_d}{T_c}\left(\frac{P_w t_w}{P_a - P_c}\right) \quad . \tag{15}$$

Once the minimum thrusting time has been chosen as a function of maximum expected disturbance torque, the achievable attitude accuracy $\pm\ \theta_d$ is determined by the zero disturbance torque limit cycle. For this case, the allowable attitude accuracy $\pm\ \theta_d$ must be at least

$$\theta_d = \frac{\dot{\theta}_0 t_{12}}{2} = \frac{T_d}{4I}\left(\frac{P_w t_w}{P_a - P_c}\right)^2 . \tag{16}$$

It is interesting to note that the controlled torque magnitude T_c is not a variable in the performance trade-off, except that it determines the firing time t_{01} for a given disturbance torque, Eq. (14).

The preceding discussion of design procedures and performance trade-offs for a single axis ion propulsion attitude control system may be extrapolated to yield three axis performance characteristics provided that cross-coupling torques between the axes can be neglected. For this simplified three-axis control case, the average coast time between successive firings may occur in different control axes. In the most conservative design, each axis is designed to accommodate the maximum disturbance torque, and the average time between firings for each axis must be at least $3\,t_{12}$. Equation (16) will then become, for the three axis case,

$$\theta_d = \frac{T_d}{4I}\left(\frac{3\,P_w t_w}{P_a - P_c}\right)^2 . \tag{17}$$

Eq. (17) may be utilized to determine the various performance trade-offs for the three-axis ion propulsion attitude control system. Figure 10 expresses these performance trade-offs in a single family of curves. From this family of curves, the achievable accuracy may be determined if the vehicle moments of inertia, maximum expected disturbance torque, and average power available are known. Conversely, for a particular design objective of average power and attitude accuracy, the required vehicle parameters may be determined.

Using these performance trade-off curves, the following fixed set of parameters are chosen to demonstrate the technique:

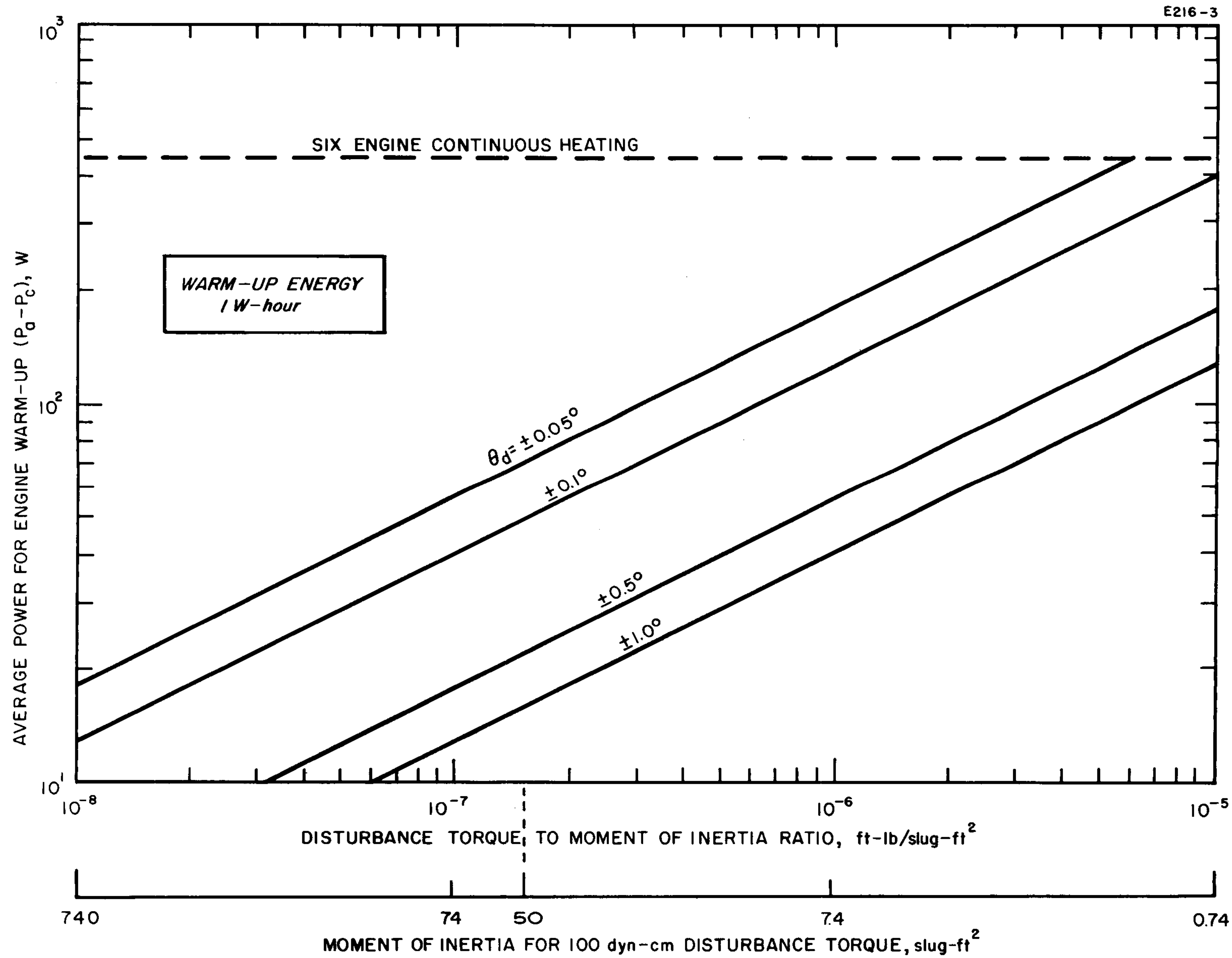

Fig. 10. Performance trade-off for three-axis attitude control system.

P_s (source output) = 55 W

P_a (power conditioning output) = 47 W

P_c (continuous power) = 25 W

$\theta_d = \pm 0.5^{\circ}$.

From Fig. 10, the ratio of T_d/I is required to be 1.5×10^{-7} or less. In the presence of a 100 dyn-cm disturbance torque, an inertia of about 50 slug-ft^2 would be required to satisfy the average power requirements. The corresponding cycle time per axis is 8.2 min.

The limit on average power requires a coast time between successive firings of at least t_{12} for both hard and soft limit cycles. While the propellant and average power consumption is greatest when the disturbance torque is either zero or the maximum design value, less power is required for intermediate values of torque. The average power consumed as a function of the disturbance torque is shown in Fig. 11.

The minimum thrusting time t_{01} in each axis for three-axis control becomes

$$t_{01} = \left(\frac{T_d}{T_c}\right)\left(\frac{3P_w t_w}{P_a - P_c}\right) . \tag{18}$$

With a control torque of 10^{-3} ft-lb, a disturbance torque of 100 dyn-cm, warmup energy of 1 W-hour, average available power of 55 W, and continuous power of 25 W, the thrusting time is 3.7 sec.

Since the control torque will, in general, be much greater than the disturbance torque, the three-axis duty cycle will be given by

$$\text{Duty Cycle} = \frac{3t_{01}}{t_{01}+t_{12}} = \frac{3T_d}{T_c} . \tag{19}$$

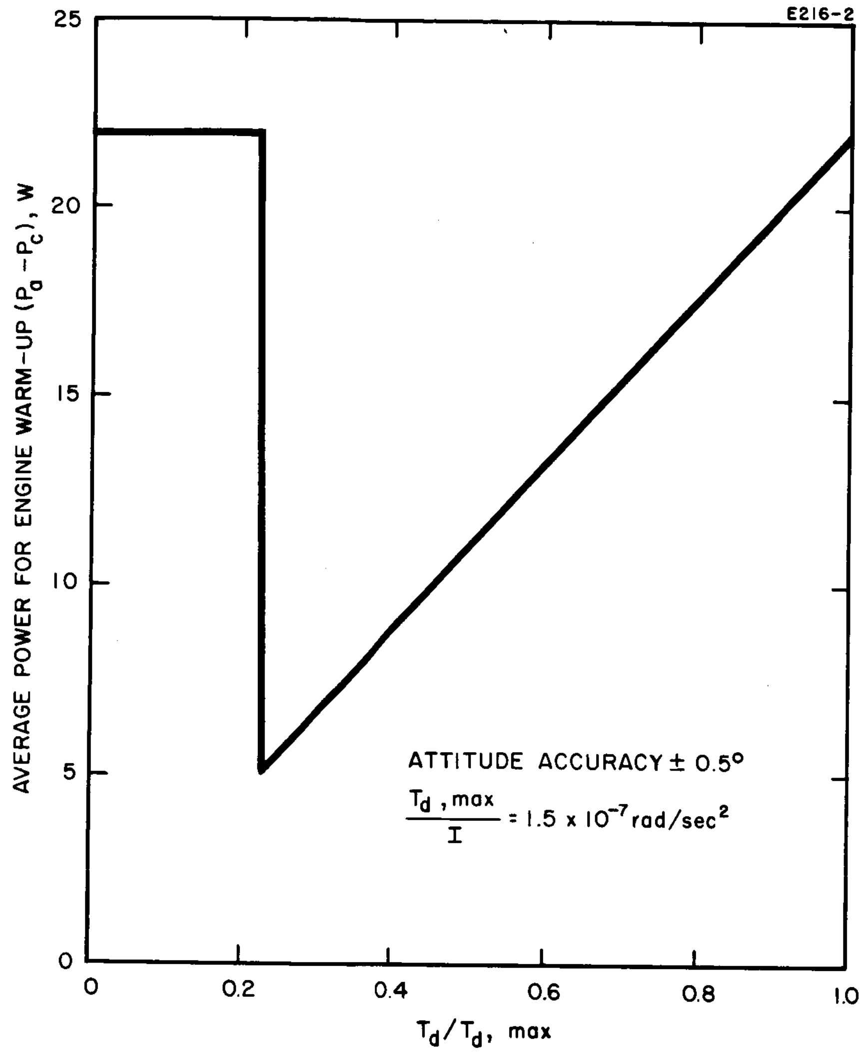

Fig. 11. Average power versus actual disturbance torque to design maximum disturbance torque ratio.

With a control torque of 10^{-3} ft-lb, the three-axis duty cycle for 100 dyn-cm disturbance torque is 2.2%. The results of this performance analysis are summarized in Table I for $\theta_d = \pm 0.05^\circ$, $P_a = 55$ W, $P_c = 25$ W, $T_c = 10^{-3}$ ft-lb, and $P_w t_w = 1$ W-hour. For purposes of comparison, the performance parameters are tabulated for three values of disturbance torque: 100, 150, and 300 dyn-cm.

TABLE I

Three-Axis Performance Parameters

Disturbance Torque T_d, dyn-cm	Minimum Moment of Inertia Required I, slug-ft^2	Coast-Time Per Axis t_{12}, sec	Thrust Time t_{01}, sec	Three Axis Duty Cycle %
100	50	492	3.7	2.2
150	75	492	5.6	3.3
300	150	492	11.1	6.6

The performance of the ion propulsion attitude control system is influenced primarily by the available average power during the coast period and the warmup power requirements of the proposed ion engine. The required vehicle inertia I will increase as a square of the average coast time allowed between firings for a given value of θ_d. Since the average coast time is directly proportional to the warmup power requirements for the ion engine, this engine characteristic should be reduced to its lowest possible value. The resulting improvement in the average power and/or the vehicle moment of inertia requirements is shown in Fig. 12 for several values of the engine warm-up energy. From Fig. 12 or Eq. (17), it can be seen that a reduction in engine warmup energy by a factor of two results in a decrease by a factor of four in the minimum inertia required or a factor of two in the average power consumption.

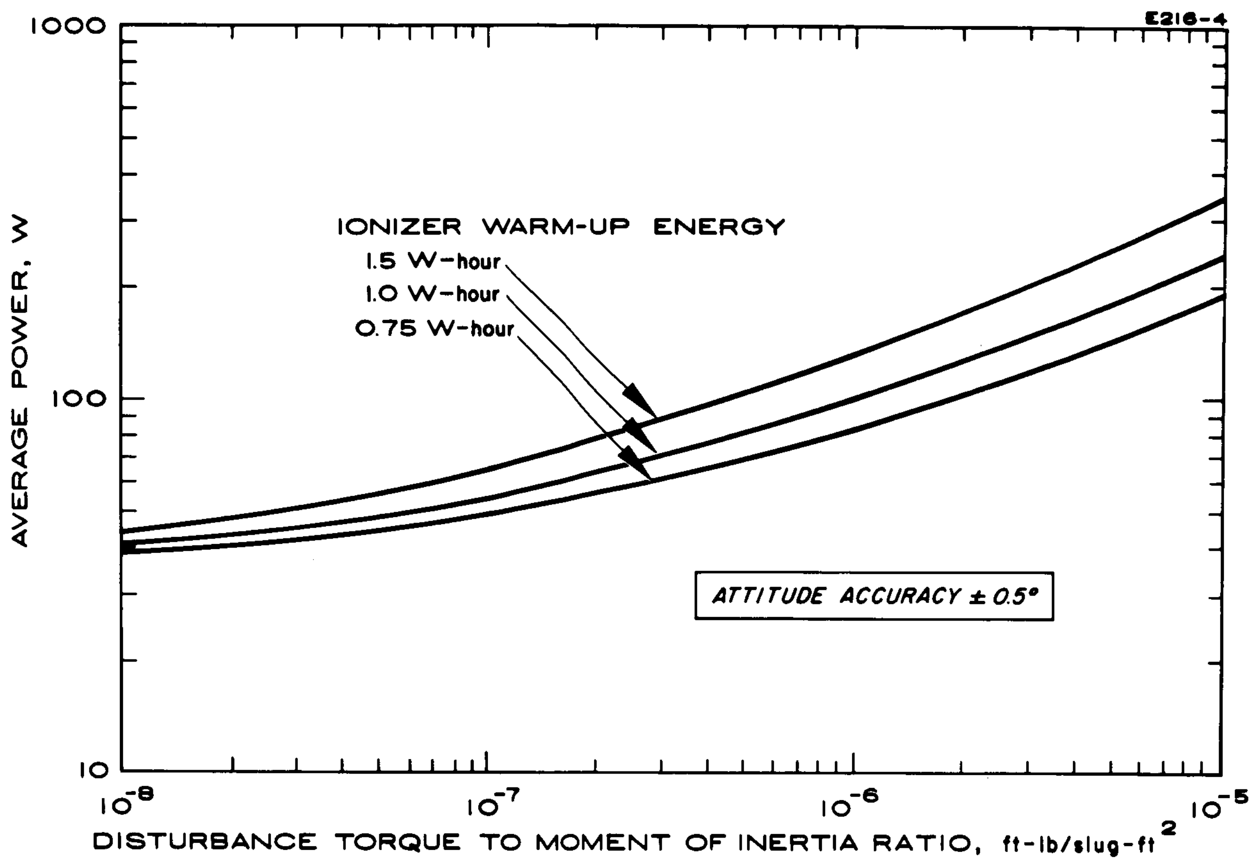

Fig. 12. Power requirement for three-axis attitude control for various ionizer warmup energies.

Figure 13 shows the average power required by the ion engine attitude control system for various values of disturbance torque (100, 150, 300, and 500 dyn-cm) as a function of vehicle moment of inertia.

3.2. Station Keeping

In Section 2.0, Mission Analysis, perturbations of the satellite orbit caused by the sun, the moon, and the ellipticity of the earth's equator are discussed, and the magnitude and direction and the sequence of the thrusting impulse required to negate these perturbations are described. The westward drift caused by the triaxiality of the earth of a satellite positioned, for example at 11° 51' E longitude (one of the points of maximum perturbation) can be negated by application of a 9.6×10^{-3} ft/sec velocity correction every 12 hours in the westward direction. In the 12-hour period that the vehicle is allowed to drift, the satellite longitude will deviate only about 1 sec off the desired 24 hour station. The magnitude of the north-south oscillations of the satellite caused by the change in orbit inclination was seen to build up a rate of 0.94° per yr. If these vehicle oscillations were left uncorrected for 37 days they would reach a peak to peak value of 0.20° in latitude (0.1° inclination). This north-south perturbation can be corrected by either of two possible thrusting modes, pulsed or continuous.

The ion propulsion station-keeping system presently under development employs three 1.5-mlb engines. Because of the relatively low thrust duty cycle required of this system, the engines when in operation may be powered by batteries, with the primary power supplied by solar cells. In order to control a 550-lb satellite, the east-west engine will fire a maximum of 1.82 min every 12 hours. The total thrust time required per correction cycle (24-hours) is shown in Fig. 14 as a function of station longitude. It can be seen by comparing Fig. 14 and Fig. 4 that the firing interval for the E-W correction will always be small compared with that for the N-S correction; thus the batteries will be sized by the latter requirement. As was shown previously, the most efficient N-S correction mode in terms of total impulse and, therefore, propellant utilization, is twice daily thrust at the node. This sequence, however, is shown in Fig. 4 to require the longest thrust interval

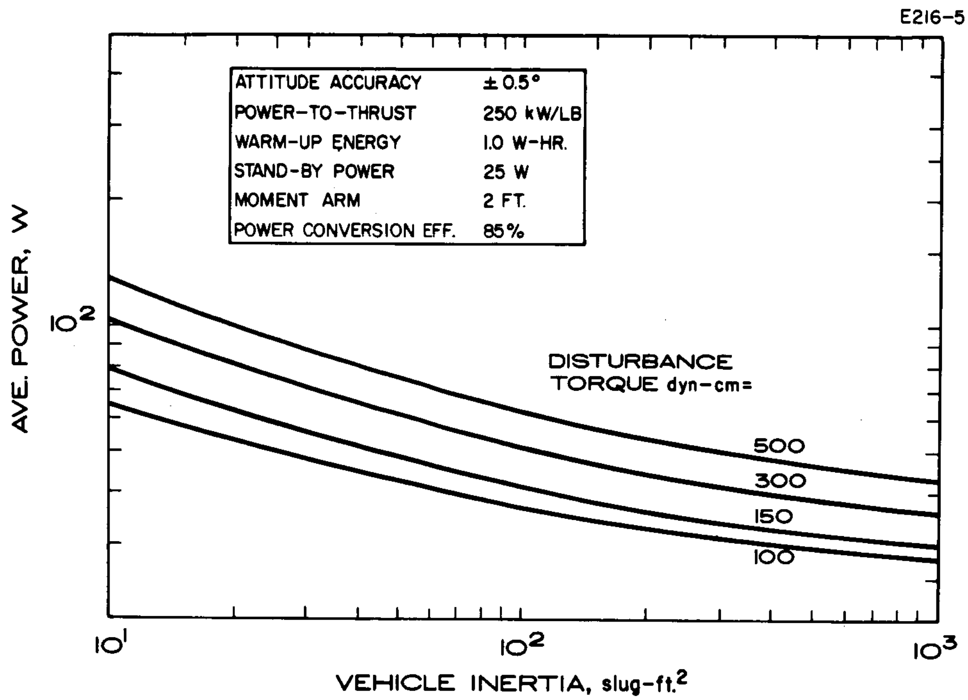

Fig. 13. Power requirement for three-axis attitude control for various disturbance torques.

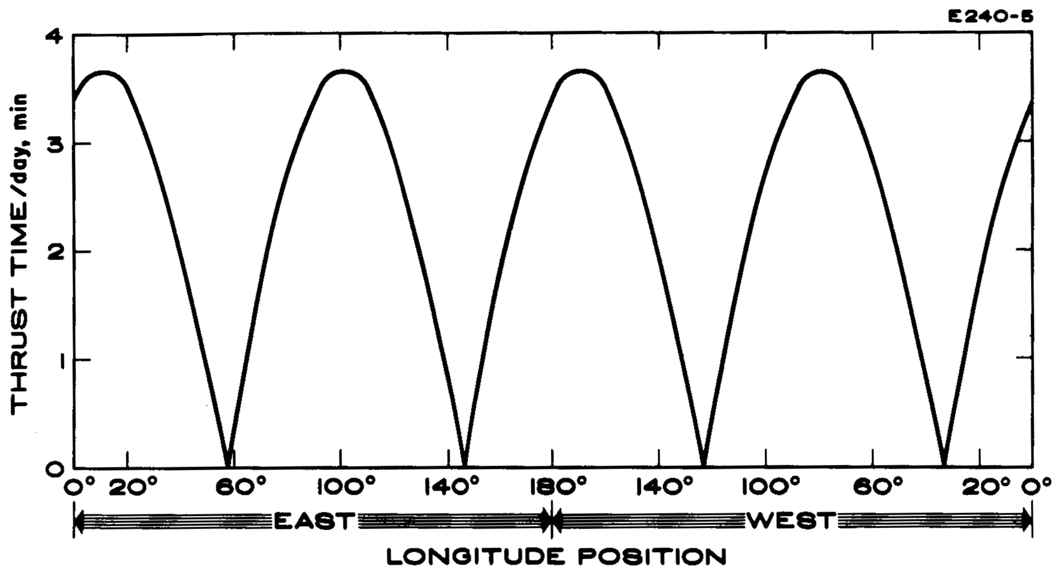

Fig. 14. Thrust time per day for E-W correction.

(largest battery). It is important, therefore, to consider alternate correction modes when optimizing the operation of this system (i.e., minimizing weight). Table II presents, along with other pertinent data, the combined power supply and propellant weight for the various thrust modes. Since for the first three modes the same basic control system is utilized, Table II shows that by thrusting six times daily in the proper positions, a weight saving of as much as 25 lb (~ 20% reduction in total system weight) can be realized.

The optimum operation of the 1.5-mlb station-keeping system in maintaining the orbit of a 550-lb synchronous satellite consists of a 24-hour correction cycle shown in Table III. The control system presently under development (MARK I) will be designed to utilize this thrust sequence.

The data presented in Table II also show the substantial weight saving afforded by operating the N-S engines in a continuous (or semicontinuous) thrust mode. Since this mode of operation requires a continuous low thrust level (0.15 mlb for a 550-lb satellite), the ion engines can be operated efficiently directly from the solar cells, thereby eliminating the need for batteries. In addition, the engine and power conditioning equipment will be somewhat reduced in weight because of the lower thrust level. It was shown in Fig. 4 that an interruption of 2 hours at the peak of the latitudinal oscillation did not noticeably reduce the effectiveness of the continuous correction mode. Since these peaks are 12 hours apart, an E-W correction can be made during these 2-hour intervals. The firing interval required to control a 550-lb satellite in the E-W direction with a 0.15-mlb engine is 18.2 min per correction (see Fig. 16). The optimum mode of operation from a weight standpoint for an ion propulsion station-keeping system is therefore a semi-continuous N-S correction at a relatively low thrust level, with the E-W correction occurring at each peak in the latitudinal oscillation. The corresponding thrust sequence for a 0.15-mlb engine system in a 550-lb satellite is presented in Table IV. It is tentatively planned that a MARK II control system be designed to operate in the manner described in Table IV.

TABLE II

North-South Station-Keeping Requirements

Thrust Mode		System Requirements*				System Weights			
Figure	Number of Thrusts Daily	Thrust Level, mlb	Average Power, W	Energy Storage, W-hour	Total Impulse Per Year lb-sec x 10^{-3}	Solar Cells, lb	Batteries, lb	Propellant, lb	Total Weight, lb
4(a)	2	1.5	29	326	2.86	3.7	46	0.64	50.3
4(b)	4	1.5	62	187	3.29	7.9	27	0.73	35.6
4(c)	6	1.5	45	135	3.55	5.7	19	0.79	25.5
4(d)	continuous	0.15	45	0	4.73	5.7	0	1.1	6.8
4(e)	semi-continuous	0.15	45	0	3.94	5.7	0	0.88	6.6

*Satellite Weight = 550 lb

TABLE III

Pulsed Mode Correction Cycle (24 Hour Period)

Engine	Thrust Position	Thrust Interval* min
North	Ascending Node	18
North	Node + 3 hours	18
East - West	Node + 6 hours	1.82
South	Node + 9 hours	18
South	Descending Node	18
South	Node + 3 hours	18
East - West	Node + 6 hours	1.82
North	Node + 6 hours	18

*Satellite Weight = 550 lb
Thrust Level = 1.5 mlb

TABLE IV

Continuous Mode Correction Cycle (24 Hour Period)

Engine	Thrust Position	Thrust Interval*
North	Symmetrical About Ascending Node	10 hours
East - West	Node + 6 hours	18.2 min
South	Symmetrical About Descending Node	10 hours
East - West	Node + 6 hours	18.2 min

*Thrust Level-to-Satellite Mass Ratio = 2.7×10^{-7} g's

In both the pulsed (6 firings daily) and continuous correction modes, the weight of the satellite to be controlled determines the weight of the power supply (i.e., solar cells and batteries). In the latter case, the satellite weight also determines the required thrust level. For example, a 0.41-mlb engine would maintain a 1500-lb satellite in the manner depicted in Fig. 4. The effect of satellite weight on power supply weight and thrust level for the two methods of north-south station keeping is shown in Fig. 15. It is seen that as the satellite weight increases, the advantage of continuous correction becomes more pronounced. The maximum thrust interval for each E-W correction is plotted in Fig. 16 as a function of satellite weight with thrust level as a parameter. The horizontal line indicates the limit of the N-S control, in terms of satellite weight, for a given thrust level. (This limit, of course, represents the continuous N-S correction mode.) From the data presented in Figs. 14 and 15, and Tables II, III, and IV, the design characteristics of the optimum station-keeping system (such as power supply requirements and weights, thrust level, duty cycle, and propellant weight) as well as the exact mode of operation can be determined for any given satellite application.

4.0. SYSTEM DESIGN

The system (Fig. 17) which is presently under development consists of eleven thrustors positioned at four stations. The stations are located on 2-ft moment arms at the extremities of the yaw and pitch axes. Three of the four control units contain two linear strip 0.5 mlb ion thrustors for attitude control and a single 1.5-mlb annular ring engine for station keeping. (A 1.5-mlb annular ion engine is shown in operation in Fig. 18.) The fourth unit contains only two linear strip engines. Gimbals are employed to permit 90° rotation of each unit, thus providing a complete redundancy in both the attitude control and station-keeping modes. As seen in Fig. 17, these gimbals are used only if one or more thrustors fail. An artist's conception of an ion engine system controlling a synchronous satellite is shown in Fig. 19.

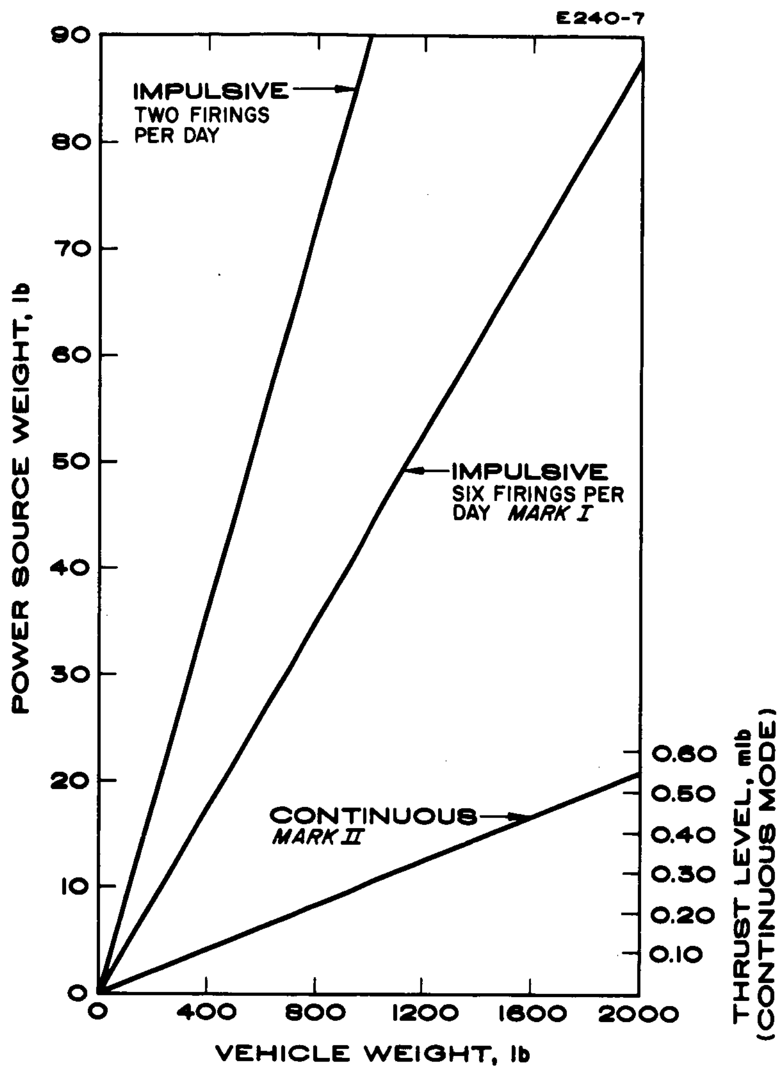

Fig. 15. Station-keeping power supply weight versus satellite weight.

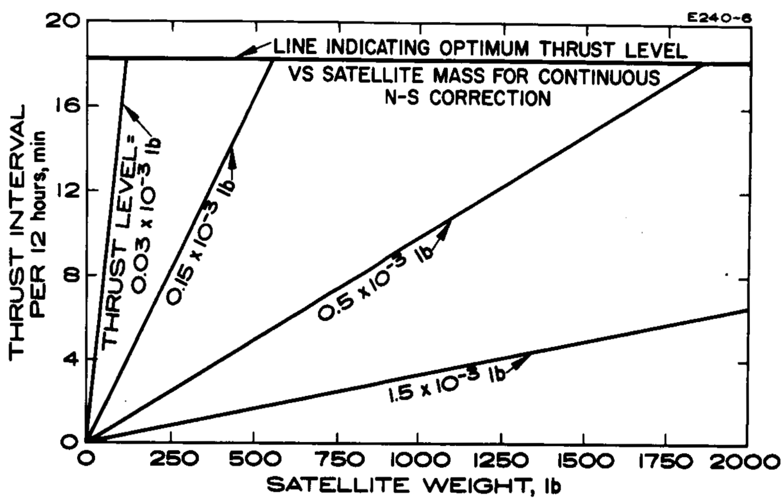

Fig. 16. Maximum thrust time per longitudinal correction versus satellite weight.

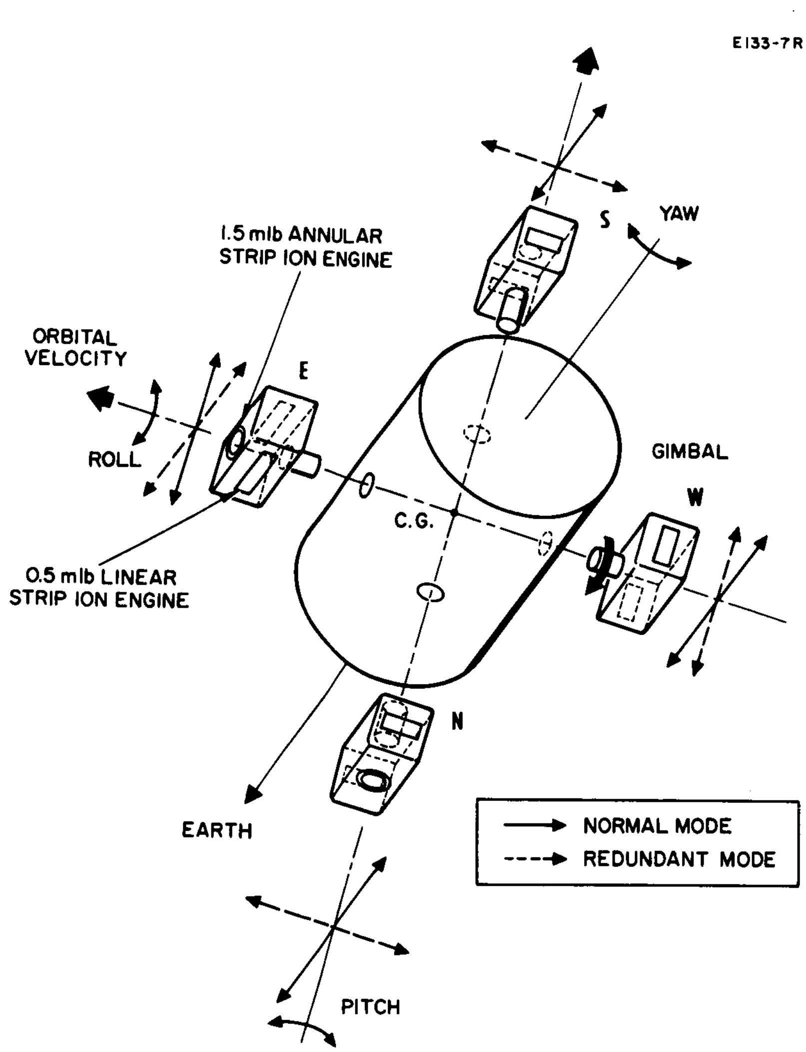

Fig. 17. Configuration of thrustors and engine stations for attitude control and station keeping system.

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Fig. 18. Photograph of 1.5-mlb annular ion engine in operation.

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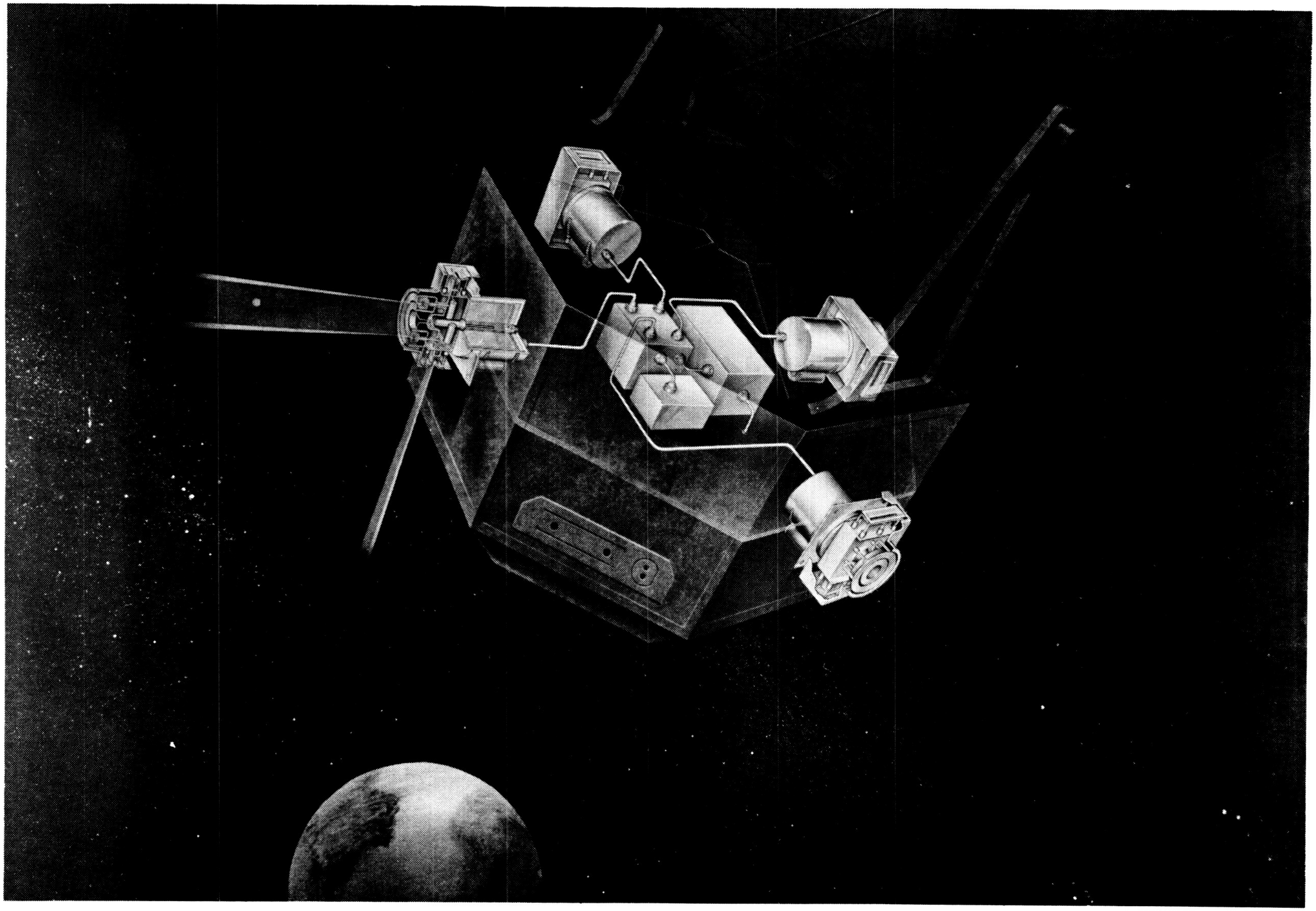

Fig. 19. Ion engine system for station keeping and attitude control of a 24-hour satellite.

In the attitude control mode, on-board sensors will supply attitude error signals in each axis. These signals are quasi-continuous so that simple shaping networks can achieve the desired rate information to insure system stability. At a predetermined signal level, as defined by the attitude accuracy desired, the porous tungsten ionizers will be heated to temperature ($\sim$ 25 sec time constant), voltage applied to the electrodes, and the ion engine fired by opening a valve in the propellant feed system. After the proper firing interval, thrust is terminated instantaneously by voltage shutdown and valve closure. The station-keeping engine will operate in a similar manner but will be controlled by on-off commands from the ground.

4.1. Engine Characteristics

In the case of attitude control, the time (and, therefore, energy) required to heat the ionizer to operating temperature is much greater than that required for thrusting. Therefore, for the attitude control engines, the largest contribution to the average power requirement is the warming up of the ionizer. The linear strip ion engines which are being developed at this time will require about 1.0 W-hour of energy to heat the ionizer. However, recent data indicate that warmup energy for a 0.5-mlb linear strip engine can be reduced to values as low as 0.75 W-hour. The effect of this improvement on system performance can be seen in Fig. 12.

Since the thrusting interval for the station-keeping mode is large compared with the ionizer warmup time, the power required to thrust is the major contribution to the average power requirement. In addition, the power needed to operate a station-keeping engine along with the thrusting time required for each north-south correction will determine (in the case of the pulsed N-S correction mode) the size and, therefore, weight of the energy storage system. For these two reasons the power to thrust ratio of the ion engines used in this application must be reduced to the lowest possible value consistent with reliability. The most efficient engines to date run at levels approaching 250 kW/lb. However, higher current density thrust devices with power to thrust ratios of 200 kW/lb are currently under development.

The choice of thrust level for both the attitude control and station-keeping engines when operating in a pulsed N-S correction mode is in a sense arbitrary. For the continuous correcting mode, however, the thrust level is determined by the weight of the satellite. In general, the lower the thrust level (for a given current density), the lower the ionizer warmup energy. For this reason, the optimum thrust level for both the attitude control engines and the station-keeping engines is that dictated by the continuous thrust mode. These lower thrust levels save valuable weight in the power conditioning equipment over higher thrust devices.

The power requirement (and, therefore, power source weight) of an ion engine of specified thrust is dependent on specific impulse. It decreases to a minimum from which point it increases monotonically with a specific impulse. The propellant weight, on the other hand, varies inversely with specific impulse. Figures 20 and 21 show the propellant weight and power source weight (station keeping only) required to maintain the orbit of a 1500-lb satellite as a function of specific impulse for the pulsed and continuous N-S correction modes, respectively. The optimum specific impulse for the pulsed mode of operation is found to be 4500 sec, whereas for the continuous correction mode the optimum specific impulse is 6500 sec.

4.2. Power Source Characteristics

In the ion propulsion attitude control and station-keeping system presently under development, it is planned that initially, at least, solar cells will be employed as the primary source of power, while batteries will provide power for the operation of the ion engine system. The total average power requirement and the solar cell weight for an ion propulsion attitude control system are shown in Figs. 22 and 23, respectively, as a function of disturbance torque to moment of inertia ratio. These curves cover a range of attitude accuracy objectives (i.e., $\pm 0.05^{\circ}$ to $\pm 1.0^{\circ}$). The vertical dotted lines represent typical maximum torque to moment of inertia ratios for satellites in the 500-lb and 1500-lb class. Figure 23 shows that the power supply weight for an ion propulsion attitude control system in the probable range of application (satellite weights between 500 lb and 1500 lb) will be less than 10 lb.

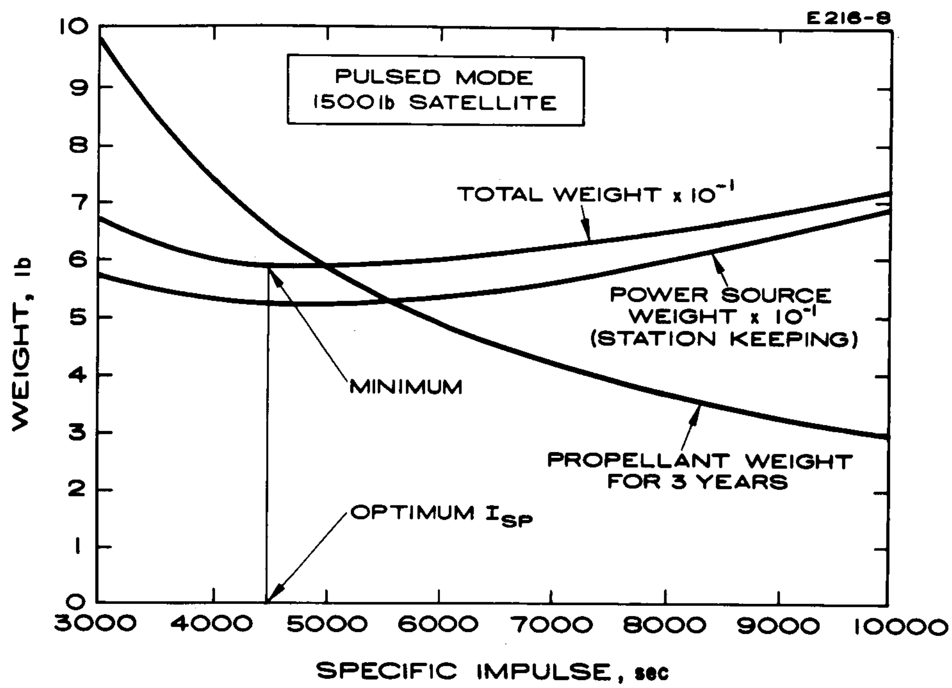

Fig. 20. System weight versus specific impulse (continuous correction mode).

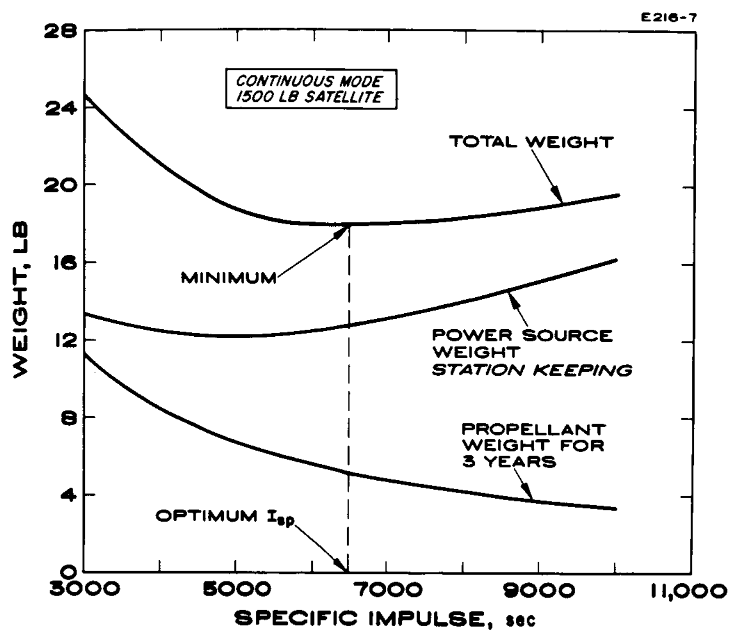

Fig. 21. System weights versus specific impulse (continuous correction mode).

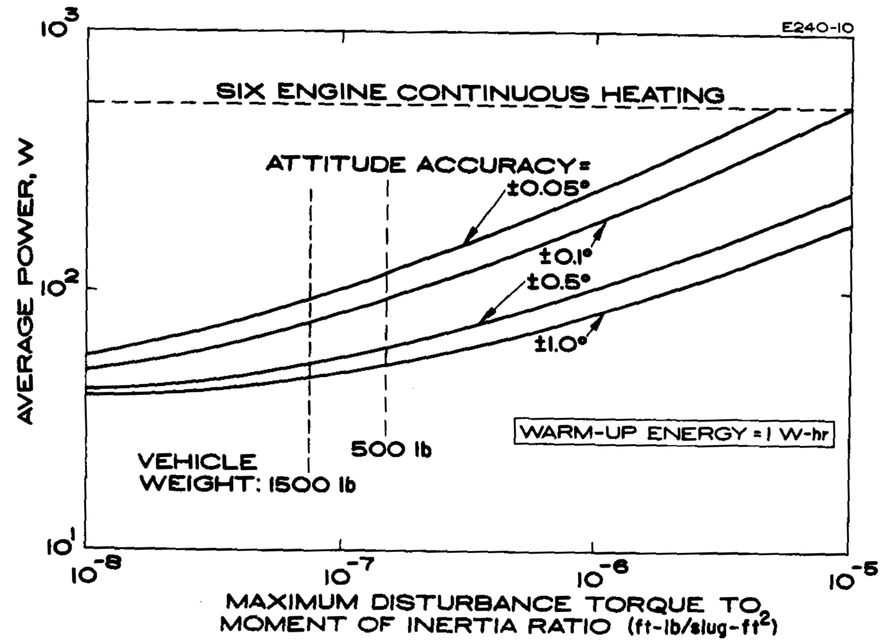

Fig. 22. Power requirement for ion engine three-axis control system.

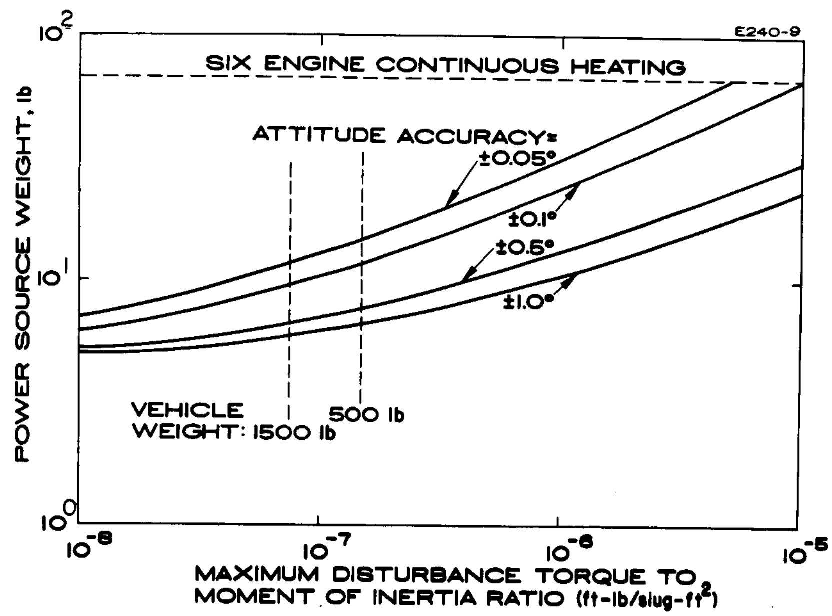

Fig. 23. Power supply weight for ion engine three-axis control system.

In the pulsed station-keeping mode, the maximum duration of single engine operation (from Fig. 4, 18 min for a 550-lb satellite or about 50 min for a 1500-lb vehicle) will determine the energy storage system size. The peak power demand (~ 900 W) occurs when one station-keeping and three attitude control engines fire simultaneously. In the continuous N-S mode of operation the engines are operated directly from the solar cells so that no energy storage system is needed for the station-keeping system. However, about 30 W-hour of energy storage capability will be required to operate the attitude control engines during the time the vehicle spends in the earth's shadow (a maximum of 1.16 hours per day). Table V shows the typical attitude control and station-keeping system characteristics required for a 1500-lb vehicle when using the continuous station-keeping thrust mode.

TABLE V

Typical System Characteristics
(1500-lb Vehicle)

Thrust Level	0.41 mlb
Power While Thrusting	123 W
Warmup Energy	1 W-hour
Average Power Requirement	175 W
Energy Storage Capability	30 W-hour
Basic Control System Weight	75 lb
Solar Cell Weight	23 lb
Battery Weight	4 lb
Total System Weight	102 lb
Propellant Weight/Year (Including Contingency)	~ 3 lb

5.0. CONCLUSION

It has been shown that the attitude control and station keeping of a stationary satellite is well within the capabilities of ion propulsion systems. Some of the problems peculiar to this application of ion engines, such as a need for extremely low ionizer warmup energy, are already near solution. Others, such as engine reliability after thousands of thrust pulses, will be accomplished and demonstrated in the near future. A prototype system of the type described in this paper will be developed for NASA at the Hughes Research Laboratories within the next year and flight hardware space tested in 1965. Subsequent to these tests, ion engine attitude control and station-keeping systems will be available for satellite missions.

Since a major portion of the system weight is contributed by the batteries required for north-south station keeping, the best pulsed mode thrust sequence for a relatively high thrust (e.g., 1.5 mlb) ion engine system is one which minimizes the individual correction intervals. It was shown, for example, that a substantial weight saving could be realized by correcting the solar-lunar perturbation six times daily. However, it was also shown that the optimum operating mode for an ion propulsion station-keeping system is continuous correction at low thrust. This latter mode of operation should therefore be used in any actual satellite application.

The capabilities of both the attitude control and station-keeping systems extend over a wide range of satellite sizes. As satellites increase in size, the ion propulsion systems become even more attractive. Figure 24 shows a comparison of the weights of several types of attitude control and station-keeping systems for use with a 1500-lb vehicle. The weights (7) of the low specific impulse cold gas, monopropellant, and bipropellant systems are for the most part dictated by propellant requirement. These weights go up rapidly as the total impulse (or mission time) increases. The weight of the ion engine system, on the other hand, is determined almost entirely by the initial hardware, and it increases very slowly ($\sim$ 3 lb/yr for a 1500-lb vehicle) with required operation time. The curves in Fig. 23 show that as the desired lifetime of satellites extends to periods of years, the ion

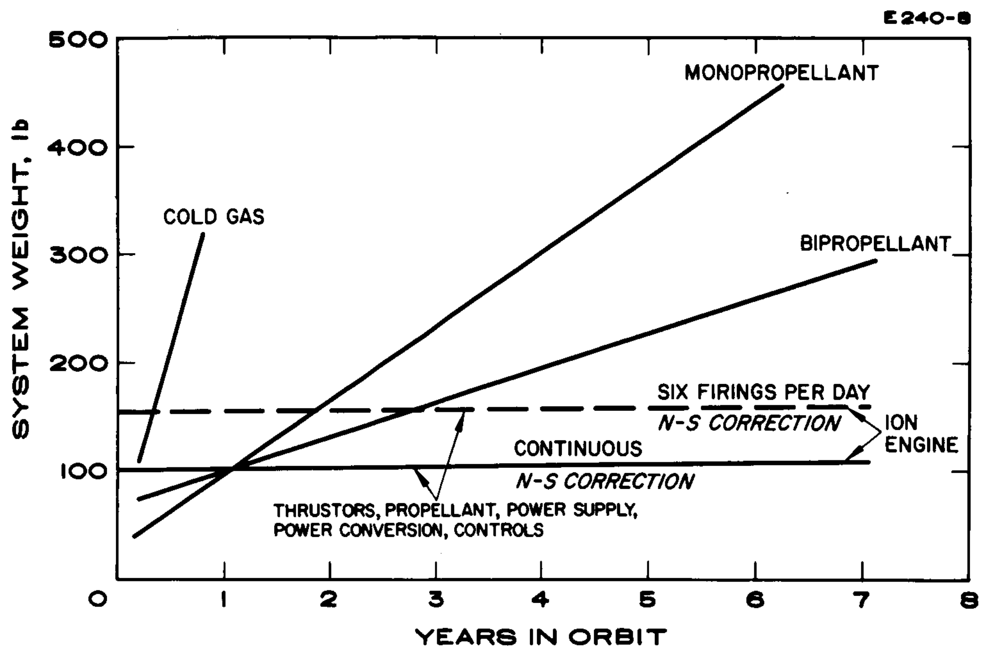

Fig. 24. Attitude control and station-keeping systems.

propulsion system emerges as the lightest attitude control and station-keeping system available. When operating in a continuous correction mode, the breaking point on weight is one year, probably the minimum desirable lifetime for communication or meteorological satellite. It can be concluded, therefore, that the ion engine systems offer both performance and weight advantages in the application of synchronous satellite control.

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